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The distribution of stress in a sandwich conical shell panel under combined load

Edward Tertel[a]*, Piotr Kuryło[a], Joanna Cyganiuk[a]

[a]*University of Zielona Góra, Faculty of Mechanical Engineering, ul. Prof. Z.Szafrana 4, 65-516 Zielona Góra, Poland*

Abstract

The paper presents an analysis of the stress distribution in the open, cone-shaped, sandwich shell panel. The shell under consideration consists of two load-carrying facings and a core layer of the soft type. The facings are made of an isotropic, compressible, work-hardening material and they are geometrically and physically symmetrical, so they are of equal thicknesses and the same material properties. The core layer resists transverse shear only. The shell under consideration is under combined external load in the form of lateral pressure and a longitudinal force. Different ratios of the longitudinal force to the lateral pressure are taken into account. In each case, the components of the effective stress as well as the effective stress values corresponding to the shell deformation were determined. To determine the stress state which occurs during the shell deformation, the stability equations have to be derived. The effective stress corresponding to the deformations in the shell under consideration can occur within the elastic, elastic-plastic or plastic range. In order to determine the stress distribution in the shell under consideration which corresponds to the on- and post-critical state, the constitutive relations of the Nadai-Hencky deformation theory, alongside the Huber-Mises-Hencky yield condition, are accepted. Analyses show that, in certain cases, there are very large differences between the values of the components of the stress state. In such cases, it seems to be possible to omit the components with small values.

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Keywords: sandwich shell; stress; stability; deformation.

* Corresponding author. Tel.: +48 68 3282577; fax: +48 68 3282497.
E-mail address: E.Tertel@ibem.uz.zgora.pl.

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1. Introduction

Thin-walled plates and shells are more often used to design and build various lightweight, modern structures, especially in transport and civil engineering. Commonly used are sandwich structures consisting of two load-carrying facings connected by an internal core-layer. The thin and flexible facings are made of a material with high strength and stiffness (sheets made of metals, plastics or composites). The core layer has a greater thickness in comparison to the facings; it is made of a less density and less rigid material (foam plastic, cork, a honeycomb or corrugated structure and others) and is stiffly placed between the facings. The result is obtaining sandwich structures with high stiffness and high compressive buckling strength, on the one hand, and comparatively low weight, on the other. Furthermore, structures of that kind have other important properties: thermal, acoustic and vibration insulation. The high structural efficiency of sandwich structures makes them very interesting for the designers and builders of lightweight, modern constructions. Those structures are well recognized in the scientific literature [1 - 4] and analyses of shells and plates under various loadings are widely presented there. Various methods of analyzing sandwich structures are presented in [1, 2] and a large reference list is given there. In some technical structures (e.g. aircraft fuselages, train bodies), conical shell elements are often used. Therefore, a three-layer conical shell panel, bi-parametrically loaded by external pressure and a longitudinal force may provide for interesting case studies. Stability and strength of thin-walled structures have been considered by many authors in the last few years. Research works refer to single-layer shells [6, 7], as well as bi-layered [8, 9] and sandwich ones [10 - 14]. In [7], some empirical results concerning conical shells subjected to external uniform pressure are presented. The proper location of the shell basic surface in the bi-layered conical shell is discussed in [8]. In [9], a derivation of the stability equation for an orthotropic elastic-plastic open conical shell is presented. Stability analyses of elastic-plastic sandwich shells with asymmetric faces are presented in [10] for cylindrical shells, and in [11-13] for conical ones.

Nomenclature

L	length of the shell
b	thickness of the facings
$2c$	thickness of the core
α	angle of inclination
β	radial angle
x, φ, z	coordinate system
N_a, N_b	longitudinal forces
q	lateral pressure
κ	ratio between the lateral and longitudinal load, $\kappa = N_a/(q \cdot x_1)$
u, v, w	displacements
w_u	deflection parameter, $w_u = w/(2(b+c))$

This article shows an analysis of the stress distribution in the open, cone-shaped, sandwich shell panel. A theoretical analysis of the stress components, as well as the effective stress distribution in the sandwich conical shell panel under combined loadings, is the main objective of this paper.

2. Object of study

The analyzed object is a sandwich conical shell panel (Fig. 1). The shell under consideration consists of two thin facings and one core layer between them, which is much thicker than the facings (Fig. 2). In this consideration, the face layers are geometrically and physically symmetrical, so they are of equal thicknesses and are made of the same material properties (steel sheet). The previous publications [11, 12] present other considerations, especially for unsymmetrical shells. The core layer is made of a lightweight material and is assumed to be elastic, incompressible in the normal *z-coordinate* direction, as well as it resists the transverse shear only. The symmetry surface of the shell is taken as the reference surface (Fig. 2).

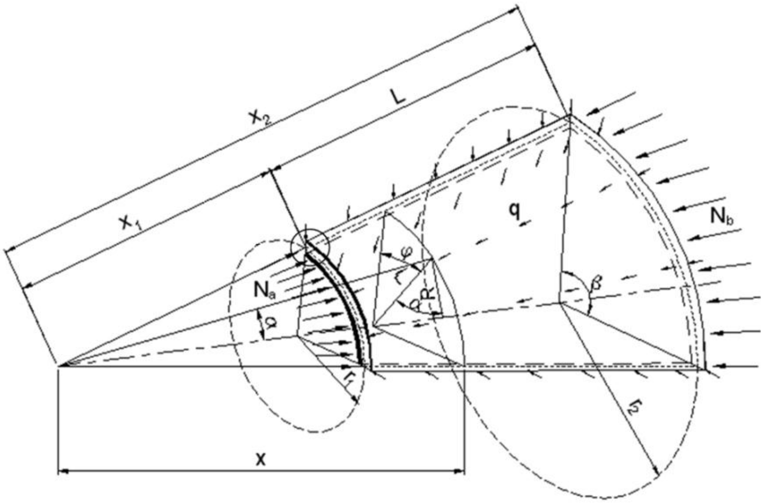

Fig. 1. The sandwich shell panel under consideration: the geometry and external loads.

There are two different types of loads acting on the shell under consideration. The first one is the surface pressure q directed perpendicularly to the shell main surface, whereas the second one represents the longitudinal forces N_a, N_b applied at the edges of the shell, (Fig. 1).

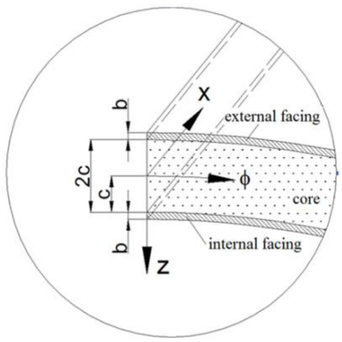

Fig. 2. The coordinate system and layer thickness.

The several assumptions, which have been taken with respect to the foregoing panel model, are as follows:

- the shell is shallow and thin-layered;
- the general theory of thin-layered shells and the geometrically nonlinear theory are obligatory;
- the elastic-plastic properties of the faces are obligatory and the bilinear stress-strain relation for the facings is accepted;
- the Kirchhoff-Love hypothesis is valid within the entire cross-section of the shell and the displacement in the normal z direction does not depend on the *z-coordinate*;
- the pre-buckling stress state is the membrane stress state;
- the post-buckling stress state can be either elastic, elastic-plastic or plastic;

- the constitutive relations of the Nadai-Hencky deformation theory with the Huber-Mises-Hencky yield condition are accepted in the analysis;
- there are no imperfections in the considered shell.

According to the previous assumptions, there is a membrane pre-buckling stress state in the shell with the internal forces described as follows:

$$N_x = \frac{1}{2} q \cdot x \cdot \mathrm{tg}(\alpha) \cdot \left[\left(\frac{x_1}{x} \right)^2 - 1 \right] - N_a \frac{x_1}{x}, \qquad N_\phi = -q \cdot x \cdot \mathrm{tg}(\alpha). \tag{1}$$

If the Kirchhoff-Love hypothesis is accepted for the facings, the relations between the displacement components: *u, v, w* of the arbitrary point of the shell and the displacements of the points situated on the middle surfaces of the specific facings u_1, u_2, v_1, v_2 are described by the following, so called broken-line, approach (Fig. 3).

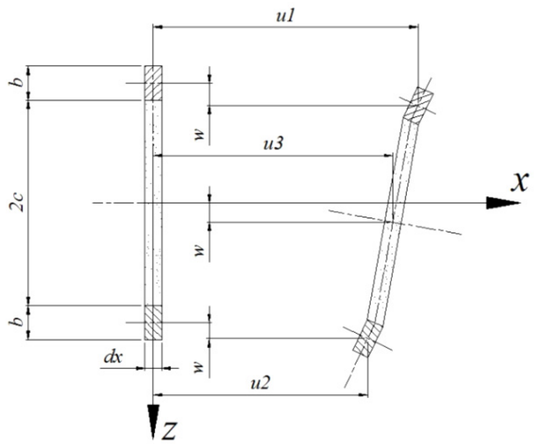

Fig. 3. The deformed shell configuration.

Introducing relations between the displacements of the points situated on the middle surfaces of the specific facings u_+, u_-, v_+, v_- [3, 11-13], the displacements for the arbitrary points of the shell could be described separately for:

- the external facing $[\,-c-b \le z \le -c\,]$:

$$u = u_+ + u_- - \left(z + c + \frac{b}{2} \right) \frac{\partial w}{\partial x}, \qquad v = v_+ + v_- - \left(z + c + \frac{b}{2} \right) \frac{1}{r} \frac{\partial w}{\partial \phi}, \tag{2}$$

- the core $[\,-c \le z \le c\,]$:

$$u = u_+ + u_- - \left(z + c + \frac{b}{2} \right) \frac{\partial w}{\partial x}, \qquad v = v_+ + v_- - \left(z + c + \frac{b}{2} \right) \frac{1}{r} \frac{\partial w}{\partial \phi}, \tag{3}$$

- the internal facing $[\,c \le z \le c+b\,]$:

$$u = u_+ + u_- - \left(z + c + \frac{b}{2}\right)\frac{\partial w}{\partial x}, \qquad v = v_+ + v_- - \left(z + c + \frac{b}{2}\right)\frac{1}{r}\frac{\partial w}{\partial \phi}, \tag{4}$$

The strains and changes in the curvature are expressed depending on the displacements as described above, using the non-linear geometrical relations given in [3].

3. Stability equations and stress distribution analysis

To calculate the effective stress and its components in the considered shell, it is necessary to derive the stability equations and calculate the critical values of the external loads (1). This goal was achieved and described in detail in the previous works [11, 12]. In those papers the stability equations of symmetrical and asymmetrical sandwich conical shells under combined load are analyzed and the procedure for obtaining the equilibrium paths and the corresponding critical loads is described. Similar methods were used and described in [10, 13] as well.

The virtual work principle and the strain energy methods comprise a basis for obtaining equilibrium equations for the considered shell. As far as the sandwich shell, the total strain energy is the sum of the strain energy of the specified layers. If the potential of external loads is represented by L_z, we have the following relations:

$$\delta U_p = \delta\left(U_w^{(1)} + U_w^{(2)} + U_w^{(3)} + L_z\right) = 0. \tag{5}$$

Where: the particular superscripts are related to the shell layers: $^{(1),\,(2)}$ – the facings, $^{(3)}$ – the core layer.
The terms in Eq.(5) relating to the strain energy were described in [11, 12].

The considered shell is free-supported. Thus, the displacement functions must satisfy the kinematic boundary conditions concerning the shell under consideration:

- the deflections at the edges of the shell are equal to zero;
- there are no displacements along the supports;
- there are no relative displacements of both layers of the shell at the edges.

Bearing in mind the above conditions, the following trigonometric displacement functions were adopted [1, 11 - 13]:

$$\begin{aligned} u_+(x,\phi) &= A_2 r^2 \cos\left(k(x - x_1)\right)\sin(p\phi), \\ u_-(x,\phi) &= A_3 r^2 \cos\left(k(x - x_1)\right)\sin(p\phi), \\ v_+(x,\phi) &= A_4 r^2 \sin\left(k(x - x_1)\right)\cos(p\phi), \\ v_-(x,\phi) &= A_5 r^2 \sin\left(k(x - x_1)\right)\cos(p\phi), \\ w(x,\phi) &= A_1 r^2 \sin\left(k(x - x_1)\right)\sin(p\phi). \end{aligned} \tag{6}$$

Where: A_i are free parameters to be determined in the solution process, k, p are constants.

Using the above displacement relations in the non-linear geometrical equations [3] describing the strains in the shell under consideration, the full description of the shell potential energy U_p is obtained [11, 12]. Next, the Ritz method was applied. This method requires that the partial derivatives of the total potential energy of the shell with respect to the parameters A_i are zero. Finally, after some transformations, a set of algebraic equations for the considered shell were obtained.

$$\begin{bmatrix} a_{11}+a_{11p} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{24} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \times \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} = \begin{bmatrix} b_{11}A_1^2 + b_{12}A_1^3 + b_{13}A_1A_2 + b_{14}A_1A_3 + b_{15}A_1A_4 + b_{16}A_1A_5 + b_{17} \\ b_{21}A_1^2 \\ b_{31}A_1^2 \\ b_{41}A_1^2 \\ b_{51}A_1^2 \end{bmatrix} \tag{7}$$

Where: a_{ij}, b_{ij} are the coefficients of the set of equations which depend on the geometrical parameters of the shell, the physical properties, the buckling form, and the external loading.

The set of equations Eqs.(7) solved with respect to the free parameter A_1 of the deflection function Eq.(6_5), following some transformations and simplifications, provides the final solution in the form of a non-linear algebraic equation:

$$q = \frac{e_1A_1 + e_2A_1^2 + e_3A_1^3}{A_1e_4\kappa + e_5} \tag{8}$$

Where: e_i are the coefficients of the stability equation and depend on the geometrical parameters, the physical properties, the buckling form and the external loads acting upon the shell, κ – the parameter which represents the ratio between the lateral and longitudinal load.

It is not possible to find an *explicite* solution to Eq.(8) because the coefficients (e_i) in this equation depend on the external loads and should be considered as a variable. The iterative methods applied in the special computer algorithm allowed for the numerical solution. The computer program makes it possible to determine the loads corresponding to the deflections of the shell under consideration. An analysis of critical loads and equilibrium paths was presented and discussed in previous works [11, 12]. The objective of this work is an analysis of the effective stress distribution, as well as the components of the effective stress distribution. To calculate the effective stress in the shell, it is necessary to calculate the values of lateral pressure q and longitudinal force N_a as functions of the deflection of the shell. Based on the values, as known, of the external loads corresponding to the deflections, we can calculate the stresses occurring in the facings of the shell. The calculations are performed for increasing values of the parameter w_u which describes the shell deflection ($w_u=w/(2(b+c))$) and for different values of the parameter κ, which represents the ratio between the lateral and longitudinal load. The stresses can, accordingly, be expressed by the external forces:

- the axial (σ_x) and circumferential (σ_φ) stress components:

$$\sigma_x = \frac{-q\cdot x\cdot \mathrm{tg}(\alpha)}{4b}\cdot\left(1-\left(\frac{x_1}{x}\right)^2\cdot\left(1-\kappa\frac{2}{\mathrm{tg}(\alpha)}\right)\right),$$
$$\sigma_\phi = \frac{-q\cdot x\cdot \mathrm{tg}(\alpha)}{2b}, \tag{9}$$

- the effective stress:

$$\sigma_i = \sqrt{\sigma_x{}^2 - \sigma_x \cdot \sigma_\phi + \sigma_\phi{}^2} = \frac{q \cdot x \cdot \mathrm{tg}(\alpha)}{4b}\sqrt{k^2 - 2k + 4},$$
$$where,\ k = \left(1 - \left(\frac{x_1}{x}\right)^2 \cdot \left(1 - \kappa\frac{2}{\mathrm{tg}(\alpha)}\right)\right). \tag{10}$$

4. Numerical results discussion

In the numerical calculations, the effective stress –Eq.(10) and its components –Eq.(9) as function of the deflection of the shell, as well as of different values of the parameter κ is performed. The input data accepted in the calculations are presented in Table 1 and Table 2.

Table 1. The geometrical parameters

Parameter	Unit	Value
Length of the shell – L	m	1,4
Smallest radius of the shell – r_1	m	2,2
Thickness of the core – $2c$	mm	16
Thickness of the facings – b	mm	0,8
Angle of inclination – α	deg.	30
Radial angle – β	deg.	30

Table 2. The physical parameters

Parameter	Unit	Value
Young's modulus (facings) – E	MPa	210 000
Tangent modulus (facings) – E_t	MPa	2,2
Poisson's ratio (facings) – ν	-	0,3
Yield stress (facings) – σ_p	MPa	0,8
Shear modulus (core) – G_3	MPa	16

The dependence of effective stress and its components on the deflection and *x-coordinate* on the shell is the basis for presentation of the results obtained and discussing them. Each of the curve on Fig. 4a represents the constant value of the parameter κ and shows the changes in the effective stress depending on the *x-coordinate* measured along the shell and for the fixed value of the shell deflection. As may be noted, the effective stress value depends on the *x-coordinate.* If the *x-coordinate* varies, the effective stress varies almost linearly.

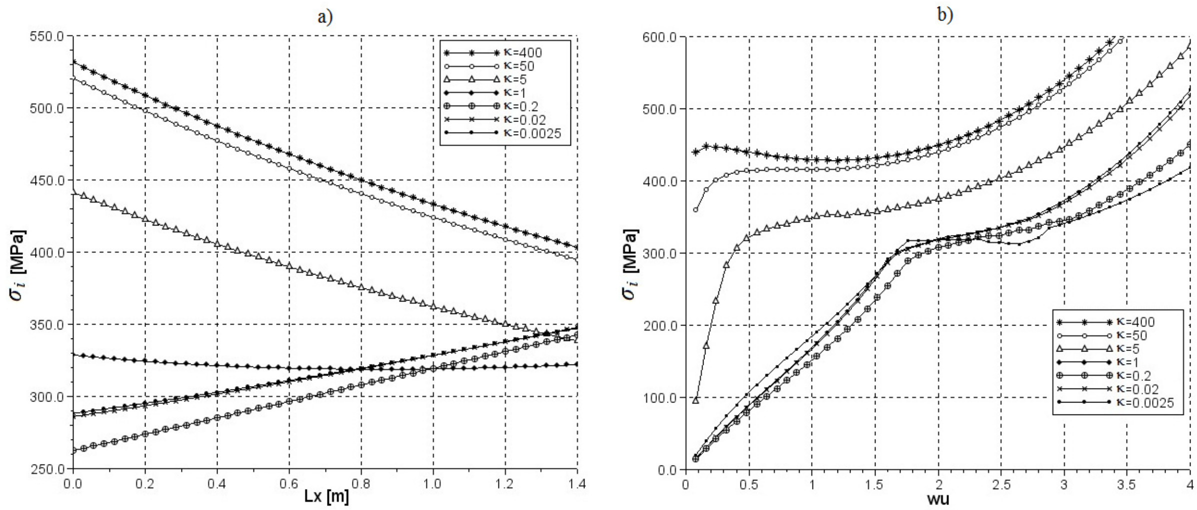

Fig. 4. The effective stress versus: (a) the x-coordinate (for $w_u = 2$), (b) the deflection (for $Lx = 1$m).

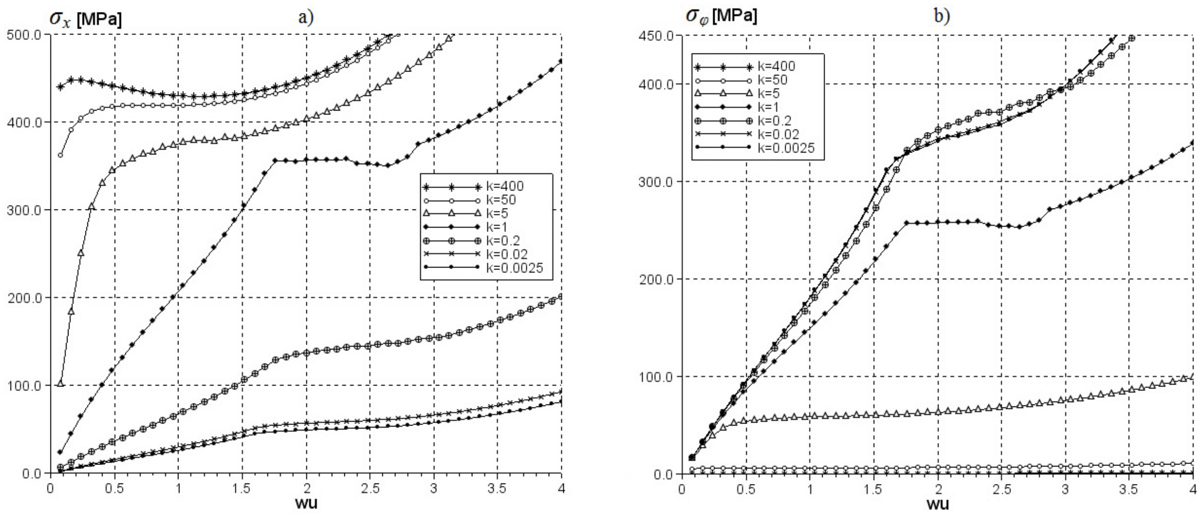

Fig. 5. The effective stress components versus the deflection (for $Lx = 1$m), (a) longitudinal stress and (b) circumferential stress.

However, the direction of changes in the effective stress values depends on the value of the parameter κ. If the parameter $\kappa < 1$, the effective stress values increases (towards the base of the cone), otherwise, if the parameter $\kappa > 1$, the effective stress drops down. For the shell panel under consideration, the value of the parameter $\kappa = 1$ is where the effective stress changes are relatively small. The curves on Fig. 4b represent the changes in the stress depending on the deflection of the shell. Two groups of the curves can be observed there. The first group corresponds to the parameter $\kappa > 5$, when the longitudinal force is the dominant type of loading. In that case, the effective stress initially increases intensively with a small increase in the deflection. Moreover, for $\kappa = 50$ or more the curves have two extreme points (maximum and minimum). In the range between the extreme points, there are instability regions where an increase of the deflection without a significant increase in the effective stress (or even with a drop in the effective stress) can be observed. The second group corresponds to the parameter $\kappa < 5$ where the surface pressure is the dominant type of loading. In that case, the effective stress increases less intensively and, for particular deflections, reaches lower values. It is noteworthy that, when the surface pressure is the dominant type of loading, the deflections in the elastic range reach bigger values in comparison to the case when the longitudinal force is the dominant type of loading.

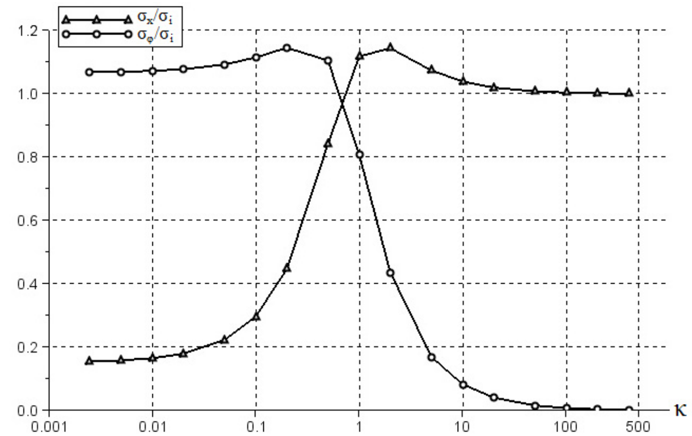

Fig. 6. The ratio of the longitudinal and circumferential stress to the effective stress.

Moreover, particular deflections correspond to the lower values of the effective stress. The curves on Fig. 5 represent the changes in the the axial (σ_x) and circumferential (σ_φ) stress components depending on the deflection of the shell. As may be noted, when the parameter κ increases, the axial stress increases too, whereas the circumferential stress decreases. If $\kappa > 50$, the values of the circumferential stress are very small in comparison to the axial stress and the effective stress. If κ =0.2 or less, the values of the circumferential stress corresponding to the particular deflection does not change significantly. The relation of the stress components to the effective stress is presented in Fig.6. As can be observed, if $\kappa > 50$, the influence of the circumferential stress on the effective stress is negligible and can be omitted in the calculation.

5. Conclusion

To summarize, it can be said that the distribution of stresses in the shell under consideration depends on the ratio of the longitudinal to lateral loads. If the longitudinal force is the dominant type of loading, the effective stress as well as the longitudinal stress reach bigger values. The effective stress exceeds the yield stress at a small deflection and deformation of the shell occurs mainly in the plastic range. If the surface pressure is the dominant type of loading, the effective stress increases less intensively during the deformation of the shell. As a consequence, the effective stress exceeds the yield stress at a bigger deflection of the shell.

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